Application of 1-D Transmultiplexer to Images Transmission

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Abstract **–The system that combines several images into a single signal appropriated for transmission by a 1-D communication channel is presented. A transmultiplexer equipped with integer-to-integer filters is applied to image transmission. Due to the incorporating of integer filters, a perfect reconstruction can be realized not only theoretically but also in practice. Such systems process signals using only the operations of multiplication and addition or subtractions of integer numbers. The new method of transmultiplexer filter design is presented. This method allows calculating separation filters arbitrary assuming composing ones. The computational algorithm, which results from a delivered theorem, and a simple example, is presented.**

I. INTRODUCTION

One of the main issues involved in the development of 4G – systems is the choice of multiple-access technology. The important task is to efficiently divide the available scarce bandwidth among a large number of users. Multiple access schemes must be spectrally efficient [1] and flexible in order to satisfy the high data rate requirement and efficient support of multimedia services. The core of 4G – system idea [2] is to create a telecommunications system enabling a wide range of hardware systems to communicate with each other and to provide a great deal of services in one network without a complicated and expensive architecture of base stations. Moreover, there should be a guarantee that any future device will be able to connect to such systems as well. The intention [3] is to create an extremely intelligent network. That is why easily reprogrammable solutions are necessary since they fulfill a need of easy implementation of new services into existing networks.

The frequency-division multiplexing (FDM) is an important method of combining signals of several users into one signal for the transmission by a single channel in currently used telecommunications systems. However, FDM has some disadvantages, one of the main problems is a very strong dependence of users data and its quality on one frequency subband. Interferences and noises occur in a narrow band, especially in radio communication. It implies a break of transmission for one or two users. It is much more convenient for telecommunications companies to spread that disturbances onto many users in the way that there will be no breaks and only a small loss of quality. Such idea was implemented in FDM by frequency hopping, but the new method with users bands independent from a single frequency is still needed. A currently introduced method – the code division multiple-access (CDMA) [1] system has probably fulfilled that important expectation. The frequency of the transmitted signal is then made to vary according to a defined pattern (code). The spectrum of each user's signal is spread over the whole channel bandwidth. It can be only intercepted by a receiver whose frequency response is programmed with the same code. The narrowband data users signal is multiplied by a spreading signal - a very large bandwidth signal. All users in a CDMA system may transmit simultaneously by using the same carrier frequency. Different users' spreading signals are approximately orthogonal to each other. The receiver performs a correlation to detect data addressed to a given user, while signals from all the other users appear as noise. The receiver needs the spreading signal used in the transmitter for detecting. The system works with uncoordinated transmission - users have no knowledge of the other users [4].

II. TRANSMULTIPLEXING

Transmultiplexers [5] are very universal systems which combine several signals into a single one. All linear methods of multiple-access are certain transmultiplexer realizations. Fig. 1 shows the traditional scheme of 1-D four-channel transmultiplexer used to transmit images by applying the Zig-Zag technique. At the transmitter side the M input images p_i^{in} are changed into 1-D signals s_i^{in} and next upsampled, filtered and summed to obtain a combined signal. This signal is sent through a single transmission channel to all recipients. At the receiver side, the combined signal is split into four channels for separation. In each channel signal is filtered, downsampled and finally transformed into image p_i^{out} to recover the original input images p_i^{in}. The basic idea is the reversibility of all procedures of transmultiplexation in such a way that all input images p_i^{in} could be recovered as precisely as possible. A transmultiplexer achieves perfect reconstruction if each image p_i^{out} is the same as image p_i^{in}, namely, if the equivalent luminance of input and output image has exactly the same value, i.e.

$$p_i^{\text{out}}(m,n) = p_i^{\text{in}}(m,n)\,, \tag{1}$$

where $i \in \{1, 2, \ldots, M\}$ is the image number and n is number of pixel in m line.

The presented system contains linear and time-invariant elements. This facilitates mathematical modelling. The

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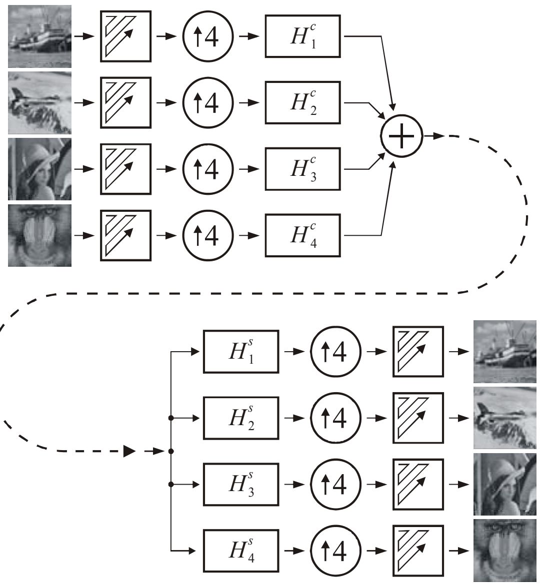

Fig. 1. A scheme of 4-channel image transmultiplexer

dependence of output s_i^{out} from inputs s_k^{in} in 1-D system, where $i,k \in \{1,2,\cdots,M\}$ is described [9] in the *z*-domain by equation

$$\bar{s}_i^{\text{out}}(z) = \frac{1}{M}\sum_{k=1}^{M} \bar{s}_k^{\text{in}}(z)\left[\sum_{m=0}^{M-1} H_k^{\text{c}}(w_M^m z^{1/M}) H_i^{\text{s}}(w_M^m z^{1/M})\right] \quad (2)$$

where $w_M = e^{-2\pi \cdot j/M}$ and $\bar{s}_i(z)$, $H^{\text{c}}(z)$, $H^{\text{s}}(z)$ stand for the z-spectrum of signal s_i, combine and separation transfer functions, respectively. It means that each output signal depends on all inputs signals. The core is that the constituent signals should be recoverable from the combined signal.

Design a transmultiplexing system means determining such coefficients for H_k^c and H_i^s filters that signals fulfil the perfect reconstruction conditions (1). Unfortunately, it is not an easy task to find [6]-[8] suitable bank of filters, although necessary and sufficient conditions given in the *z*-domain are known [9]. It is needed to solve the bilinear equations which result from the perfect reconstruction condition and the mathematical model (2) of the transmultiplexer. There are various possible methods to solve this issue. One can use numerical procedures to determine filter coefficients by minimizing appropriate defined quantity criterion [7][8]. Sometimes it is possible to obtain solutions for simple systems. In general, however, there are no known methods to solve the obtained set of equations. What is more, it is difficult even to examine the existence and uniqueness of solutions. It is generally known that if filter orders are sufficiently large, there are solutions and there are many of them. This paper is an attempt to overcome these difficulties and moreover to apply 1-D transmultiplexer system to simultaneous transmission of several images through a single 1-D telecommunication channel.

Looking for solutions leads to determining the sufficient conditions that give equations possible to solve. Such equations give only some solutions. Procedure of this kind is presented in this paper below. Sufficient conditions are given in the form of theorem and its proof is presented in [10].

III. DESIGN OF FILTER BANKS

Let us limit filter design to the case of FIR filters of low orders *I*. Let $h_i^s(0) = 0$ for all $1 \le i \le M$ and moreover for $p = 1,\dots,m$ let us introduce matrices

$$G_p^{\text{c}} = \begin{bmatrix} h_1^{\text{c}}(pM-1) & h_1^{\text{c}}(pM-2) & \cdots & h_1^{\text{c}}(pM-M) \\ h_2^{\text{c}}(pM-1) & h_2^{\text{c}}(pM-2) & \cdots & h_2^{\text{c}}(pM-M) \\ \cdots & \cdots & \cdots & \cdots \\ h_M^{\text{c}}(pM-1) & h_M^{\text{c}}(pM-2) & \cdots & h_M^{\text{c}}(pM-M) \end{bmatrix} \quad (3)$$

$$G_p^{\text{s}} = \begin{bmatrix} h_1^{\text{s}}(pM-M+1) & \cdots & h_M^{\text{s}}(pM-M+1) \\ h_1^{\text{s}}(pM-M+2) & \cdots & h_M^{\text{s}}(pM-M+2) \\ \cdots & \cdots & \cdots \\ h_1^{\text{s}}(pM) & \cdots & h_M^{\text{s}}(pM) \end{bmatrix} \quad (4)$$

which consist of coefficients of combine and separation filters, respectively. Matrices (3) and (4) are square and their dimensions depend on the number of channels

$$G_p^{\text{c}}, G_p^{\text{s}} \in \Re^{M\times M}. \quad (5)$$

Causal filters make that a certain delay τ is unavoidable. The admissible delay satisfies inequalities

$$1 \le \tau \le \left\lfloor \frac{2I}{M} \right\rfloor - 1, \quad (6)$$

where operation $\lfloor . \rfloor$ returns the greatest integer number equal to or less then the argument. This means that the maximal admissible delay is proportional to the range of filters *I* and inversely proportional to the number of channels *M*.

Theorem [10]:

Let $m+1 \le \tau \le 2m-1$ for arbitrary taken natural number *m*. If $\det G_m^{\text{c}} \ne 0$ and

$$\begin{aligned} &G_1^{\text{c}} = \ldots = G_{m-1}^{\text{c}} = 0,\ G_1^{\text{s}} = \ldots = G_{\tau-m}^{\text{s}} = 0, \\ &G_{\tau-m+1}^{\text{s}} = (G_m^{\text{c}})^{-1}, \end{aligned} \quad (7)$$

then conditions of perfect reconstruction are satisfied.

For the case when $\det G_m^{\text{s}} \ne 0$ under assumptions

$$\begin{aligned} &G_1^{\text{s}} = \ldots = G_{m-1}^{\text{s}} = 0,\ G_1^{\text{c}} = \ldots = G_{\tau-m}^{\text{c}} = 0, \\ &G_{\tau-m+1}^{\text{c}} = (G_m^{\text{s}})^{-1}, \end{aligned} \quad (8)$$

the perfect reconstruction is obtained as well.

□

Theorem suggests the method of filters design. Some coefficients of composition filters must be assumed and the relations (7) enable us to compute some parts of separation filters. The remaining values of filters coefficients are equal to zero. It is possible to proceed in an opposite direction, some parts of separation filters can be assumed and the parameters of composition filters (8) should be computed. Algorithm presented below result from the first part of theorem presented above.

Algorithm

According to the theorem, matrices G_2^c and G_2^s must be non-singular. Let us assume $m = 2$, then we obtain a simple algorithm for filter designing:

- substitute $G_1^s = G_1^s = 0$
- arbitrarily choose matrix G_2^c (i.e. coefficients of composition filters),
- calculate $G_2^s = (G_2^c)^{-1}$,
- according to (3) and (4) use matrices $G_1^c, G_2^c, G_1^s, G_2^s$ to obtain filter coefficients.

It is always possible to provide calculations using rational numers only. To obtain the integer calculations it is sufficient to take integer elements in G_2^c and fulfil condition $\det G_2^c = 1$. Integer coefficients are highly desirable because of their fast and low-powered VLSI implementations. The integer signal representation is a necessary condition for lossless compression.

IV. EXAMPLE

The simple example of a *M*-channel filters with integer coefficients can take the Walsh-Hadamard transform values. The method presented in this paper enables us to design filters with more irregular coefficients. For example, the algorithm presented above under assumption

$$G_2^c = \begin{bmatrix} 1 & -1 & 2 & -2 \\ -1 & 2 & -3 & 4 \\ 2 & -3 & 6 & -7 \\ -2 & 4 & -7 & 10 \end{bmatrix} \tag{9}$$

gives the following combine filters

$$h_1^c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -2 \\ 2 \\ -1 \\ 1 \end{bmatrix} \quad h_2^c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ -3 \\ 2 \\ -1 \end{bmatrix} \quad h_3^c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -7 \\ 6 \\ -3 \\ 2 \end{bmatrix} \quad h_4^c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 10 \\ -7 \\ 4 \\ -2 \end{bmatrix} \tag{10}$$

and the separation filters

$$h_1^s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 1 \\ -2 \\ -1 \end{bmatrix} \quad h_2^s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 3 \\ 0 \\ -1 \end{bmatrix} \quad h_3^s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \quad h_4^s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}. \tag{11}$$

This example presents integer-to-integer filter banks. Integer values for the combine filter coefficients were assumed and the integer values were obtained for the separation filter coefficients.

Input images have to be serialized to obtain 1-D signals before transmultiplexing. The standard methods of serialization of images use the row or column order. Applying of such an order causes rapid changes of amplitude for inter-line changes. It is caused by very small correlation of pixels on the end of the line and on the beginning of the next line. Very high frequencies appear in such 1-D signal. It is better to apply a sequence ZIG-ZAG to set the order of 2-D signal serialization. In this method only pixels in the closest range are classified. This is why their luminances do not differ too much, unless there are sharp edges in the image. Images turned into 1-D signals present lowpass properties (see Fig.2). The harmonic components other than low frequencies are suppressed by at least 15 dB and around of 90% of the band is below 50 dB. It gives amplitude more than 300 times smaller than amplitude of constant component.

The composite signal is an integer one and in the range of –1712 and 2463 because of filtering and summing up of the component images. The combined signal spectrum (see Fig. 3) is concentrated around constant component and around sampling frequency f_s and its value multiplied by two. This feature is typical for transmultiplexers. The multiplication of the spectrum is caused by upsampling procedures.

V. CONCLUSION

For the next-generation telecommunications, the goals are the high data rates and customisable applications enabled by flexible technologies. Carefully designed transmultiplexers are able to fulfill such needs. Wideband wireless communications techniques have many advantages, however, there are a number of challenges and new possibilities in that area [11].

A transmultiplexer is a multiple-access system which arranges an easy adaptation and co-operation with existing telecommunications systems. It assimilates the idea of an intelligent network and reprogrammable electronic devices. Different linear systems can be introduced by a change of filter coefficients only, without hardware modifications.

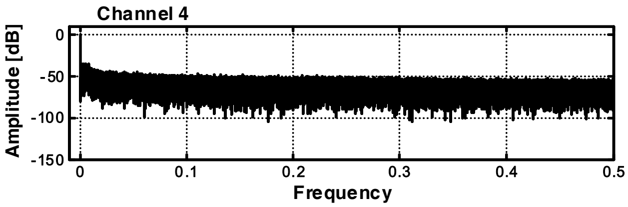

Fig. 2. Spectrum of signal obtained by applying the ZIG-ZAG technique to tested image

Filtering, upsampling and downsampling can be implemented in any kind of digital equipment. In the case where the coefficients of FIR filters are approximated via the least square method, they had a finite precision occurrence, so some errors were unavoidable. Perfect reconstruction conditions are only theoretically fulfilled because of the finite precision of digital computations. Practically, output signals were a little different from input ones. To transmit uncoded images or audio signals such results were satisfactory, but usually there is a different situation. Software files or coded multimedia data such as MPEG files, even with very slight changes, are damaged and impossible to use. The application of integer-to-integer filter banks increases the number of possible applications. Methods enabling the calculation of integer coefficients for perfect reconstruction filters are particularly useful. Transmultiplexing systems equipped with such filters process signals using only the operations of multiplication and addition or subtractions of integers. Therefore, there are no computation errors such as rounding of numbers that occur in operations on real numbers. Owing to this, transmultiplexing systems equipped with integer filters can be used not only to transmit multimedia signals but also for encrypted data or lossless compressed images where a change in even one single bit is inadmissible.

VI. ACKNOWLEDGEMENT

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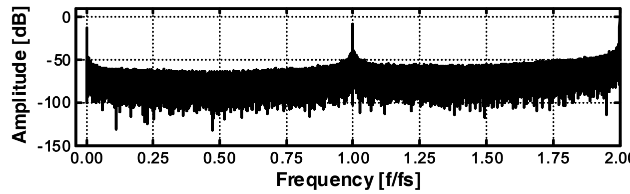

Fig. 3. The spectrum of transmitted signal

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