Robust Stability of D-symmetrizable Hyperbolic Systems

by

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Abstract. The systems under consideration are governed by a set of first-order linear partial differential hyperbolic equations together with boundary conditions. The Lyapunov method is used to verify the stability of the initial-boundary value problem. Necessary and sufficient conditions for stability are obtained under the assumption that the matrix coefficients in the differential equations and in the boundary conditions are D-symmetrizable. The considered systems have an interesting property: Hurwitz type stability and Schur type stability occur in one system simultaneously. The stability of the continuous type system is a stability of wave propagation. The stability of the discrete type system is a stability of the boundary feedback and the boundary reflections. Necessary and sufficient conditions for the robust stability of an initial-boundary value problem are obtained for the case where the matrix coefficients belong to a convex hull of stable and D-symmetrizable matrices.

Key words: robust stability, hyperbolic equations, Hurwitz stability, Schur stability, symetrizable matrices

1. Introduction. The stability of a family of systems has frequently been investigated in recent publications (see for example Białas [1], Rohn [8,9], Qian and DeMarco [10], Cohen and Lewkowicz [3], Wang *et al* [12], Keel and Bhattacharyya [5]). The first results were published by Kharitonov in 1978 for the robust Hurwitz stability of polynomials. From that time extensive research has been directed toward extending these results to other systems. Research in these areas is generally directed towards computationally efficient algorithms for

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desirable locations for roots of polynomials or eigenvalues of matrices when polynomials or matrices vary over a family.

Komentarz [MZ1]:

The obtained results are connected with the finite dimensional systems described by ordinary differential equations (continuous-time systems), or difference equations (discrete-time systems). This paper considers the robust stability of systems described by the set of first order partial differential equations of the hyperbolic type with boundary conditions posed in proper form. We limit the discussion to linear and stationary systems. The stability of these dynamic systems depends on the location of the eigenvalues of coefficient matrices: for partial differential equations (Hurwitz type stability) and for boundary conditions (Schur type stability). For the systems presented in this paper, both types of stability occur simultaneously. The mathematical model, presented in Section 2, describes in the usual way, the phenomenon of wave propagation and wave reflection (e.g. the distributed electric network presented by Ziółko [13]). This type of equation also describes some types of mass or energy transportation.

The application of the Lyapunov functional method to investigating the stability of hyperbolic systems with nonzero boundary conditions is presented in Section 3. We used an energy functional associated with a diagonal matrix. This confines the class of problems for which obtaining necessary and sufficient conditions for asymptotic stability is feasible to the D-symmetrizable systems. It is typical of hyperbolic systems that computational difficulties confine considerations only to symmetric (e.g. Russell [11]) or symmetrizable systems (see Ziółko [13]).

The method of examining robust stability through robust nonsingularity is applied in Section 4. Necessary and sufficient conditions for Hurwitz robust stability as well as Schur robust stability are obtained for D-symmetrizable matrices.

2. Hyperbolic Systems. Consider the canonical form of a linear system consisting of first order partial differential equations

(1) $$\frac{\partial y}{\partial t} + \Lambda \frac{\partial y}{\partial x} = A\,y$$

where $\Lambda, A \in \Re^{nxn}$ are constant matrices. Partial differential equation (1) is the hyperbolic type if matrix Λ has a complete set of eigenvectors and its eigenvalues are real. Without loss of generality matrix Λ can be transformed to the diagonal form

(2) $$\Lambda = \begin{bmatrix} \Lambda^- & 0 \\ 0 & \Lambda^+ \end{bmatrix} \qquad \begin{matrix} \Lambda^- < 0 \\ \Lambda^+ > 0 \end{matrix}$$

in such a way that elements on the diagonal of Λ satisfy the inequalities

(3) $$-\infty < \lambda_1 \le \lambda_2 \le \cdots \le \lambda_p < 0 < \lambda_{p+1} \le \lambda_{p+2} \le \cdots \le \lambda_n < \infty .$$

The nonzero solution of equation (1) can be uniquely determined for

(4) $$\Psi = \{(x,t) : 0 \le x \le 1,\, t \ge 0\}$$

if initial values are specified

(5) $$y(x,0) = y_0(x) \qquad \text{for} \qquad 0 \le x \le 1$$

and boundary conditions are assumed

(6) $$\begin{bmatrix} y^-(1,t) \\ y^+(0,t) \end{bmatrix} = B \begin{bmatrix} y^-(0,t) \\ y^+(1,t) \end{bmatrix} \qquad \text{for} \quad t \ge 0$$

where the dependent variable

$$y(x,t) = \begin{bmatrix} y^-(x,t) \\ y^+(x,t) \end{bmatrix} \in \Re^n$$

is divided into two parts: $y^-(x,t) \in \Re^p$ and $y^+(x,t) \in \Re^{n-p}$ corresponding to the partition of matrix Λ. Constant block matrices A and B consist of the four matrices

(7) $$A = \begin{bmatrix} A_{--} & A_{+-} \\ A_{-+} & A_{++} \end{bmatrix}, \quad B = \begin{bmatrix} B_{01} & B_{11} \\ B_{00} & B_{10} \end{bmatrix} \qquad \begin{matrix} A_{-+}, B_{00} \in \Re^{(n-p)xp} & A_{++}, B_{10} \in \Re^{(n-p)x(n-p)} \\ A_{--}, B_{01} \in \Re^{pxp} & A_{+-}, B_{11} \in \Re^{px(n-p)} \end{matrix} .$$

The matrices B_{00} and B_{11} describe the boundary reflections for the boundary at $x=0$ and $x=1$, respectively. The matrices B_{10} and B_{01} describe the feedback from the boundary $x=1$ to the boundary $x=0$ and from the boundary $x=0$ to the boundary $x=1$, respectively.

If $y_0(x)$ is continuously differentiable and satisfies the consistency conditions

$$\begin{bmatrix} y_0^-(1) \\ y_0^+(0) \end{bmatrix} = B \begin{bmatrix} y_0^-(0) \\ y_0^+(1) \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} -\Lambda^- \dfrac{d\,y_0^-}{d\,x}(1) + A_{--}y_0^-(1) + A_{+-}y_0^+(1) \\ -\Lambda^+ \dfrac{d\,y_0^+}{d\,x}(0) + A_{-+}y_0^-(0) + A_{++}y_0^+(0) \end{bmatrix} =$$

$$B \begin{bmatrix} -\Lambda^- \dfrac{d\,y_0^-}{d\,x}(0) + A_{--}y_0^-(0) + A_{+-}y_0^+(0) \\ -\Lambda^+ \dfrac{d\,y_0^+}{d\,x}(1) + A_{-+}y_0^-(1) + A_{++}y_0^+(1) \end{bmatrix} \tag{9}$$

then solution $y(x,t)$ is in $C^1(\Psi)$. Consistency conditions (8), (9) constitute the C^1 regularity for the points where the initial conditions meet with the boundary conditions. Similar regularity conditions and more details about the regularity of solution $y(x,t)$ are presented by Courant and Hilbert [4] and Russell [11].

3. Stability of Hyperbolic Systems.

DEFINITION 1. The linear homogeneous dynamic system (1), (6) is asymptotically stable if

$$E(t) = \|y(\cdot,t)\|^2 \to 0 \text{ as } t \to \infty. \tag{10}$$

Property (10) can be verified if such a norm $\|\cdot\|$ is known that the derivative dE/dt exists and its sign is constant for all values of variable *y*. If *dE/dt<0* for all *y* then we conclude that system (1), (6) is asymptotically stable. In the opposite case, if $dE/dt > 0$ for all *y*, the considered system is unstable. Thus we obtain sufficient conditions for the stability or instability of system (1), (6). Combining these two conditions, it is sometimes possible to obtain sufficient and necessary conditions for asymptotic stability. The main problem consists in finding a monotone norm (10). The stability and the asymptotic behavior of the solutions of differential equations was considered in a similar way by Lakshmikantham and Leela [7] on pp. 175-179.

Let the energy of system (1), (6) be defined as the square of $L^2(0,1)$ norm

(11) $$E(t) = \|y(\cdot,t)\|_G^2 = \int_0^1 y^T(x,t)\,G\,y(x,t)\,dx$$

where $G \in \Re^{nxn}$ is a fixed positive definite matrix.

The first derivative of the energy functional (11) for the C^1 regular vector function y is given by

(12) $$\frac{dE}{dt} = \int_0^1 \left(\frac{\partial y^T}{\partial t} G\,y + y^T G \frac{\partial y}{\partial t} \right) dx\,.$$

Substituting

(13) $$\frac{\partial y}{\partial t} = Ay - \Lambda \frac{\partial y}{\partial x}$$

and using the identity

(14) $$\int_0^1 \frac{\partial y^T}{\partial x} \Lambda G y\,dx = y^T \Lambda G y\Big|_0^1 - \int_0^1 y^T \Lambda G \frac{\partial y}{\partial x}\,dx$$

from (12) we obtain

(15) $$\frac{dE}{dt} = \int_0^1 y^T \left(A^T G + GA\right) y\,dx - y^T \Lambda G y\Big|_0^1 + \int_0^1 y^T \left(\Lambda G - G\Lambda\right) \frac{\partial y}{\partial x}\,dx\,.$$

To establish the sign of dE/dt we must make an additional assumption. If matrix G is diagonal, then the matrices Λ and G commute, and the bilinear component in formula (15) vanishes. Due to this restriction on matrix *G*, the stability conditions presented below are obtained under the assumption that matrices *A* and *B* are D-symmetrizable.

DEFINITION 2 [2]. Matrix $A = a_{ij} \in \Re^{nxn}$ is D-symmetrizable, if there exists a diagonal matrix

(16) $$K = diag\left(k_1, k_2, \ldots, k_n\right) \in \Re^{nxn}$$

such that $k_i > 0$ *(i=1,2,...,n)* and equation $KA = A^T K^T$ is satisfied.

It is well-known that in hyperbolic equations (1), which are of great importance in applied mathematics, matrices *A* fulfill the postulate of D-symmetrizability. It is a matter of natural laws: if some quantity influences another quantity then the latter has a similar effect on the former quantity. Therefore the matrix *A* in the hyperbolic partial differential equation (1) is usually D-symmetrizable. It is quite a different story with the matrix *B* in boundary condition (6). The phenomenon, which occurs at one boundary, can not be similar to the phenomenon which occurs at the other boundary. Therefore the matrices *B* may be non-D-symmetrizable in mathematical models of processes which occur in practice.

Taking into account boundary conditions (6), we obtain for diagonal matrix *G*

(17) $$\frac{dE}{dt} = \frac{dE_i}{dt} + \frac{dE_b}{dt}$$

where

(18) $$\frac{dE_i}{dt} = \int_0^1 y^T\left(A^T G + GA\right) y\, dx$$

(19) $$\frac{dE_b}{dt} = \begin{bmatrix} y^-(0,t) \\ y^+(1,t) \end{bmatrix}^T \left[B^T |\Lambda G| B - |\Lambda G|\right] \begin{bmatrix} y^-(0,t) \\ y^+(1,t) \end{bmatrix}$$

and $|\Lambda G|$ is the positive and diagonal matrix whose elements are the absolute values of the respective elements of matrix ΛG. By this method the first derivative (15) of the energy functional (11) was divided into two parts: (18) and (19). The first part depends on the matrix

A and the second part depends on matrix B. Matrix A is the coefficient of differential equation (1) which describes the dynamics of the system in the interior of the set Ψ. Matrix B is the coefficient of the boundary conditions (6) and describes the properties of variable y at the boundaries of the set Ψ.

DEFINITION 3. The initial-boundary value problem (1), (5), (6) is asymptotically interior *stable* if there exists a positive definite diagonal matrix G such that the inequality $dE_i/dt < 0$ holds for all $t \geq 0$, and all nonzero initial conditions (5).

Let us denote $\lambda_1(A), \lambda_2(A), \ldots, \lambda_n(A)$ as the eigenvalues of the matrix $A = [a_{ij}] \in \Re^{nxn}$.

THEOREM 1. *If matrix $A \in \Re^{n \times n}$ from equation (1) is D-symmetrizable (by matrix K^2) then the initial-boundary value problem (1), (5), (6) is asymptotically interior stable if and only if all the eigenvalues of A are negative, i.e. $\lambda_i(A) < 0$ for $i = 1, 2, \ldots, n$.*

P r o o f. Substituting $G = K^2$ and $y = K^{-1}\eta$ we obtain

(20) $$y^T\left(A^T G + GA\right)y = 2\eta^T KAK^{-1}\eta .$$

This quadratic form is negative definite if and only if all the eigenvalues of A are negative.

DEFINITION 4. The initial-boundary value problem (1), (5), (6) is asymptotically *boundary stable* if there exists a positive definite diagonal matrix G such that $dE_b/dt < 0$ for all $t \geq 0$ and for arbitrary y not equal to zero at both boundaries simultaneously.

THEOREM 2. *If matrix $B \in \Re^{n \times n}$ in equation (6) is D-symmetrizable (by matrix K^2), then the initial-boundary value problem (1), (5), (6) is asymptotically boundary stable if and only if the magnitudes of all eigenvalues of B are less than 1, i.e. $|\lambda_i(B)| < 1$ for $i = 1, 2, \ldots, n$.*

P r o o f. The inequality $dE_b/dt < 0$ is equivalent to

(21) $$y^T\left[B^T |\Lambda G| B - |\Lambda G|\right]y < 0 \qquad \text{for every } \; y \in \Re^n .$$

The D-symmetrizability of matrix B leads to the equation $KBK^{-1} = K^{-1}B^{T}K$. This means that matrix B is similar to the symmetric matrix KBK^{-1} and the eigenvalues of B are real. Substituting $|\Lambda G| = K^2$ and $\eta = Ky$, we obtain condition (21) in the form

(22) $$\frac{\eta^T \left(KBK^{-1}\right)^2 \eta}{\eta^T \eta} < 1 .$$

This Rayleigh quotient is less than 1 for arbitrary $\eta \neq 0 \in \Re^n$ if and only if the magnitudes of all eigenvalues of the matrix B are less than 1.

LEMMA 1. *If the initial-boundary value problem (1), (5), (6) is asymptotically as well interior as boundary stable then* $E(t) = \|y(\cdot,t)\|^2 \to 0$ *as* $t \to \infty$.

P r o o f. Assumption $dE_b/dt \le 0$ converts equation (17) into the inequality

(23) $$\frac{dE}{dt} \le \frac{dE_i}{dt} .$$

Let us denote the greatest eigenvalue of the matrix G as $\lambda_{\max}(G)$. From (18) we have

(24) $$\frac{dE_i}{dt} \le \lambda_{\max}(A^T G + GA)\, \|y\|_I^2 \le -\tau\, \|y\|_G^2$$

where $\tau = -\dfrac{\lambda_{\max}(A^T G + GA)}{\lambda_{\max}(G)}$ and $\tau > 0$ since $\lambda_{\max}(A^T G + GA) < 0 < \lambda_{\max}(G)$. Combining inequalities (23) and (24) we obtain

(25) $$\frac{dE}{dt} \le -\tau\, E(t) .$$

This leads to the inequality

(26) $$E(t) \le E(0)\, e^{-\tau t} \quad \text{for } t > 0 .$$

According to the definitions of interior and boundary stability, these two phenomena occur independently although they are incorporated in one system. The result is that for a system, which is interior stable and boundary unstable (i.e. $\exists\, G : dE_i/dt < 0,\, dE_b/dt > 0$ for all nonzero initial conditions), we know nothing about the stability or instability of the whole system. For this case it is always possible to choose C^1 initial conditions such that $\|y(0,0)\|_{l^2}$ and $\|y(1,0)\|_{l^2}$ are sufficiently large and $\|y(x,0)\|_{l^2}$ is small enough for every $x \in (0,1)$ to obtain $dE/dt = dE_i/dt + dE_b/dt > 0$ for t=0. Because $E(t)$ is a C^1 continuous function of *t*, there exists $\varepsilon > 0$ such that $dE/dt > 0$ for $0 \leq t < \varepsilon$. This observation suggests that the system is unstable, although it is possible that $E(t) \to 0$ as $t \to \infty$ for all initial conditions. On the other hand, from interior stability and boundary stability (with common *G*) it follows that the whole system is stable. It is possible however, to find a system which is interior stable (there exists a diagonal matrix $G_i > 0$ such that $dE_i/dt < 0$ for all nonzero initial conditions); and boundary stable (there exists a diagonal matrix $G_b > 0$ such that $dE_b/dt < 0$ for all nonzero boundary values); but for which there is no diagonal matrix $G > 0$ such that $dE_i/dt < 0$ and $dE_b/dt < 0$ simultaneously for all $t \geq 0$. All these problems follow from the fact that the Lyapunov method gives sufficient conditions for the stability of dynamic systems and only sometimes (for special kinds of systems) it gives the necessary conditions as well. In our case the D-symmetrizability of matrices *A* and *B*, and the splitting (17) of the derivative of the Lyapunov function into two parts (18) and (19), enables us to obtain not only sufficient but also the necessary conditions in Theorem 1 and Theorem 2.

4. Robust Stability of D-symmetrizable Matrices. Let

(27) $$\mathbf{S}_H = \left\{ A \in \Re^{nxn} : \mathrm{Re}\big(\lambda_i(A)\big) < 0,\ i = 1,2,\ldots,n \right\}$$

(28) $$\mathbf{S}_S = \left\{ A \in \Re^{nxn} : |\lambda_i(A)| < 1,\ i = 1,2,\ldots,n \right\}$$

be the set of Hurwitz and the set of Schur stable matrices, respectively.

DEFINITION 5. SYM is a set of D-symmetrizable matrices if every matrix $A \in SYM \subset \Re^{n \times n}$ is D-symmetrizable.

THEOREM 3. *If* $P = [p_{ij}], Q = [q_{ij}] \in \Re^{nxn}$ *and*

1) P, Q are Hurwitz stable matrices,

2) *for every matrix* $A \in \mathbf{S} = \{\alpha P + (1-\alpha) Q : \alpha \in [0,1]\}$ *all eigenvalues are real, i.e.*

$$\lambda_i(A) \in \Re, \quad (i = 1, 2, \ldots, n)$$

*then the set **S** is Hurwitz stable (i.e.* $\mathbf{S} \subset \mathbf{S}_H$*) if and only if*

$$\lambda_i(PQ^{-1}) \notin (-\infty, 0] \qquad (i = 1, 2, \ldots, n) \tag{29}$$

P r o o f. From assumption (29) it follows that

$$\det\left(PQ^{-1} - \lambda I\right) \neq 0 \quad \text{for every} \quad \lambda \in (-\infty, 0] \tag{30}$$

where $I \in \Re^{nxn}$ is the unit matrix. Substituting $\lambda = \frac{\alpha - 1}{\alpha}$ we obtain from (30)

$$\det\left(\alpha P + (1-\alpha) Q\right) \neq 0 \quad \text{for every} \quad \alpha \in (0,1]. \tag{31}$$

Taking into account assumption 1) and 2) we obtain $S \subset S_H$ from (31) because the eigenvalues of $\alpha P + (1-\alpha) Q$ are continuous functions of parameter α.

To prove the necessary condition, from $S \subset S_H$ we conclude that (31) holds. From the stability of matrix Q it follows that Q^{-1} exists, thus (31) can be written in form (30).

Usually, it is difficult to verify assumption 2) of Theorem 3. Only the sufficient conditions for real eigenvalues of matrices are known. One such example is the case of D-symmetrizable matrices (for more details see [2]).

COROLLARY 1. If the assumptions of Theorem 3 hold and matrices P and Q span the set of D-symmetrizable matrices (i.e. $S \subset SYM$ see Białas and Ziółko [2]), then problem (1), (5), (6) is robust interior stable, i.e.

(32) $$\mathbf{S} = \left\{\alpha P + (1-\alpha)Q : \alpha \in [0,1]\right\} \subset \mathbf{S}_H \cup SYM$$

THEOREM 4. *If* $P = [p_{i,j}]$, $Q = [q_{i,j}] \in \Re^{nxn}$ *and*

1) matrices P, Q are Schur stable,

2) *for every* $B \in \mathbf{S} = \left\{\alpha P + (1-\alpha)Q : \alpha \in [0,1]\right\}$ *it holds* $\lambda_i(B) \in \Re$, *(i=1,2,...,n) then*

the set ***S*** *is Schur stable (i.e.* $\mathbf{S} \subset \mathbf{S}_S$ *) if and only if*

(33) $$\lambda_i\left((I-P)(I-Q)^{-1}\right) \notin (-\infty,0] \text{ and } \lambda_i\left((I+P)(I+Q)^{-1}\right) \notin (-\infty,0] \text{ for } i=1,2,...,n.$$

P r o o f. (If) From assumption (33) we obtain

(34) $$\det\left((I-P)(I-Q)^{-1} - \lambda I\right) \neq 0 \text{ and } \det\left((I+P)(I+Q)^{-1} - \lambda I\right) \neq 0 \text{ for } \lambda \in (-\infty,0].$$

These conditions are equivalent to

(35) $$\det\left(I - P - \lambda(I-Q)\right) \neq 0 \text{ and } \det\left(I + P - \lambda(I+Q)\right) \neq 0.$$

Substituting $\lambda = \frac{\alpha - 1}{\alpha}$ to (35) we obtain for $\alpha \in (0,1]$

(36) $$\det\left(\alpha P + (1-\alpha)Q - I\right) \neq 0 \text{ and } \det\left(\alpha P + (1-\alpha)Q + I\right) \neq 0.$$

This means that -1 and 1 are not eigenvalues of matrices $\alpha P + (1-\alpha)Q$. Additionally, taking into account 1) and 2) we obtain finally

(37) $$\left|\lambda_i\left(\alpha P + (1-\alpha)Q\right)\right| < 1 \quad \text{for } i = 1,2,\dots,n \text{ and all } \alpha \in [0,1]$$

because the eigenvalues of matrices $\alpha P + (1-\alpha)Q$ are continuous functions of parameter α.

To prove the necessary condition we notice that $S \subset S_S$ leads to (36). From the Schur stability of matrix Q we obtain $\det\left(Q - \lambda I\right) \neq 0$ for $\lambda \leq -1$ or $\lambda \geq 1$. This means that $\left(I-Q\right)^{-1}$ and $\left(I+Q\right)^{-1}$ exist and it enables us to obtain (34) from (36).

COROLLARY 2. If the assumptions of Theorem 4 hold and matrices P and Q span the set of D-symmetrizable matrices (i.e. $S \subset SYM$) then problem (1), (5), (6) is robust boundary stable, i.e.

(38) $$\mathbf{S} = \left\{ \alpha P + (1-\alpha) Q : \alpha \in [0,1] \right\} \subset \mathbf{S}_S \cup SYM$$

5. Conclusions. We have presented the problem of the robust stability of a distributed parameter system which is described by a set of first order partial differential equations of the hyperbolic type. It appeared that stability investigations led to the problems well known for systems with lumped parameters: the Hurwitz type stability and the Schur type stability. It is interesting that both these types of stability occur in system (1), (6) simultaneously. To obtain the necessary and sufficient conditions for robust stability the considerations are limited to D-symmetrizable systems. The main results are presented in Corollary 1 and Corollary 2.

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