Transmultiplexer Integer-to-Integer Filter Banks

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***Abstract* - Transmultiplexers equipped with integer-to-integer filters can be applied for transmitting coded signals or computer data. The new method of transmultiplexer filter design is presented.**

***Keywords* - transmultiplexer, multiple-access, filter bank.**

I. INTRODUCTION

One of the main issues involved in the development of B3G systems is the choice of multiple-access technology. The important task is to efficiently divide the available scarce bandwidth among a large number of users. Multiple access schemes must be spectrally efficient [1] and flexible in order to satisfy the high data rate requirement and efficient support of multimedia services. The core of 4G-system idea [2] is to create a telecommunications system enabling a wide range of hardware systems to communicate with each other and to provide a great deal of services in one network without a complicated and expensive architecture of base stations. Moreover, there should be a guarantee that any future device will be able to connect to such systems as well. The intention [3] is to create an extremely intelligent network. That is why easily reprogrammable solutions are necessary since they fulfil a need of easy implementation of new services into existing networks.

According to mathematical models, transmultiplexers [4] are very universal systems. All linear methods of multiple-access are certain transmultiplexer realizations. Unfortunately, it is not an easy task to find correct bank of filters [5,6,7], although necessary and sufficient conditions given in the *z*-domain are known [8]. This paper is an attempt to overcome these difficulties.

II. TRANSMULTIPLEXING

Transmultiplexer combines several signals into a single one. Fig.1 shows the traditional scheme of a four-channel transmultiplexer. At the transmitter side the *M* input signals are upsampled, filtered and summed to obtain a composite signal. This signal is sent through a single transmission channel to all recipients. At the receiver side, the composite signal is split into four channels for separation. The signal is filtered and downsampled to obtain again the original input signals. The presented system contains linear and time-invariant elements. This facilitates mathematical modelling.

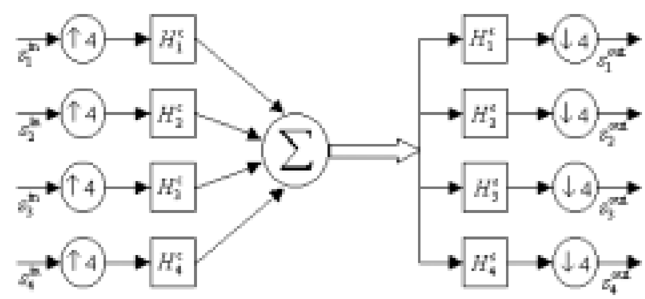

Fig. 1. A scheme of 4-channel transmultiplexer.

The basic idea is the reversibility of all procedures of transmultiplexation in such a way that all output signals s_i^{out} could be recovered as precisely as possible. A transmultiplexer achieves perfect reconstruction if s_i^{out} is only a delayed version of s_i^{in}, namely, if there exists a positive integer τ such that

$$s_i^{\text{out}}(n) = s_i^{\text{in}}(n-\tau), \tag{1}$$

where $i \in \{1, 2, \ldots, M\}$ is the signal number.

The dependence of output s_i^{out} from inputs s_k^{in}, where $i, k \in \{1, 2, \cdots, M\}$ is described [2] in the *z*-domain by equation

$$\bar{s}_i^{\text{out}}(z) = \frac{1}{M}\sum_{k=1}^{M} \bar{s}_k^{\text{in}}(z)\left[\sum_{m=0}^{M-1} H_k^{\text{c}}(w_M^m z^{1/M})\, H_i^{\text{s}}(w_M^m z^{1/M})\right], \tag{2}$$

where $w_M = e^{-2\pi \cdot j/M}$ and $\bar{s}_i(z)$, $H^c(z)$, $H^s(z)$ stand for the *z*-spectrum of signal s_i, combining and separation transfer functions, respectively. It means that each output signal depends on all inputs signals. The core is that the constituent signals should be recoverable from the combined signal. To achieve the perfect reconstruction (1), under assumption that all filters are FIR type of order *I*, from (2) we obtain a set of equations

$$\sum_{l=\max\{0,Mn-I\}}^{\min\{Mn,I\}} h_k^c(Mn-l)\, h_i^s(l) = M\delta_{k,i}\delta_{n,\tau}, \tag{3}$$

where $\delta_{k,i}$ is a Kronecker function, $h_i^s(l)$ means the *l*-th coefficient of filter $H_i^s(z)$, $i \in \{1,2,\dots,M\}$. Similarly, $h_k^c(Mn-l)$ stands for the coefficients of the $H_k^c(z)$ filter. For each pair $i,k \in \{1,2,\cdots,M\}$ of filter numbers, condition (3) gives $[2I/M]+1$ equations for $n = 0,1,\dots,[2I/M]$, where $[2I/M]$ means an integer part of 2I/M. It results in the system of $M^2([2I/M]+1)$ equations (3). The admissible delay fulfils inequalities

$$1 \le \tau \le \left[\frac{2I}{M}\right] - 1.$$

It means that a certain delay is unavoidable and its maximal value is proportional to the range of filters *I* and inversely proportional to the number of channels *M*.

Designing a transmultiplexing system means determining such coefficients for H_k^c and H_i^s filters that fulfil conditions (3). This is a difficult task due to the fact that set (3) consists of bilinear equations. There are various possible methods to solve this issue. One can use numerical procedures to determine filter coefficients by minimizing the residua obtained from equations (3). Sometimes it is possible to obtain solutions for simple systems. In general, however, there are no known methods to solve a bilinear set of equations and therefore (3) is not useful in designing digital filters. What is more, it is difficult even to examine the existence and uniqueness of solutions of equations (3). It is generally known that if filter orders are sufficiently large, there are solutions and there are many of them.

Looking for solutions leads to determining the sufficient conditions that can be written in a more simple form. Such equations give only some certain solutions of (3). Procedures of this kind are presented in this paper below. Sufficient conditions are given in the form of theorems. Proofs are not presented here. Computational algorithms resulting from theorems and simple examples are presented.

III. Designing Of Filter Banks

We will focus on the case when filters have low orders. For a certain $m \in N$, let the orders of all combining filters *K* be sufficiently large and depend on the number of channels in the following way: $K \le mM - 1$. Let the order of all separation filters be $L \le mM$ and moreover $h_i^s(0) = 0$ for all $1 \le i \le M$. For $p = 1,\dots,m$ let us introduce matrices

$$G_p^c = \begin{bmatrix} h_1^c(pM-1) & h_1^c(pM-2) & \cdots & h_1^c(pM-M) \\ h_2^c(pM-1) & h_2^c(pM-2) & \cdots & h_2^c(pM-M) \\ \cdots & \cdots & \cdots & \cdots \\ h_M^c(pM-1) & h_M^c(pM-2) & \cdots & h_M^c(pM-M) \end{bmatrix} \tag{4}$$

$$G_p^s = \begin{bmatrix} h_1^s(pM-M+1) & \cdots & h_M^s(pM-M+1) \\ h_1^s(pM-M+2) & \cdots & h_M^s(pM-M+2) \\ \cdots & \cdots & \cdots \\ h_1^s(pM) & \cdots & h_M^s(pM) \end{bmatrix} \tag{5}$$

which consist of coefficients of combining and separation filters, respectively. Matrices (4) and (5) are square and their dimensions depend on the number of channels

$$G_p^c, G_p^s \in \Re^{M\times M}. \tag{6}$$

Theorem 1.
Let $1 \le \tau \le m$. If $\det G_1^c \ne 0$, then filters which satisfy conditions $G_1^s = \dots = G_{\tau-1}^s = 0$,

$$G_\tau^s = (G_1^c)^{-1}, \tag{7}$$

fulfil the perfect reconstruction conditions (3). If $\det G_1^s \ne 0$, then filters which satisfy $G_1^c = \dots = G_{\tau-1}^c = 0$,

$$G_\tau^c = (G_1^s)^{-1}, \tag{8}$$

fulfil conditions (3) as well.

Theorem 2.
Let $m+1 \le \tau \le 2m-1$. If $\det G_m^c \ne 0$ and

$$G_1^c = \ldots = G_{m-1}^c = 0,\; G_1^s = \ldots = G_{\tau-m}^s = 0,$$

$$G_{\tau-m+1}^s = (G_m^c)^{-1}, \quad (9)$$

then conditions (3) are satisfied. For the case $\det G_m^s \neq 0$ and

$$G_1^s = \ldots = G_{m-1}^s = 0,\; G_1^c = \ldots = G_{\tau-m}^c = 0,$$

$$G_{\tau-m+1}^c = (G_m^s)^{-1}, \quad (10)$$

conditions (3) are satisfied as well.

Theorem 3.
Let $\tau \in \{1,\ldots,m\}$. If $\det G_1^c \neq 0$, then matrices which fulfil the following conditions: $G_1^s = \ldots = G_{\tau-1}^s = 0$, $G_\tau^s = (G_1^c)^{-1}$,

$$G_{\tau+1}^s = -(G_1^c)^{-1} G_2^c (G_1^c)^{-1},$$

$$G_2^c \left((G_1^c)^{-1} G_2^c\right)^{m-1} = \left((G_1^c)^{-1} G_2^c\right)^m = 0,$$

$$G_k^c = G_2^c \left((G_1^c)^{-1} G_2^c\right)^{k-2} = G_1^c \left((G_1^c)^{-1} G_2^c\right)^{k-1} \quad \text{for} \quad k = 3,\ldots,m,$$

consist of filter coefficients which are solutions of (3).

Let us define matrix

$$G = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 1 \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \in \Re^{M \times M}$$

and let its k-th power be denoted by G^k.

Theorem 4.
Let $\tau \in \{1,\ldots,M\}$. For arbitrary matrix G_1^c such that

$$\det G_1^c \neq 0,$$

$$G_k^c = G_1^c G^{k-1} \quad (11)$$

for $2 \leq k \leq M$, $G_i^s = 0$ if $1 \leq i \leq \tau - 1$,

$$G_\tau^s = (G_1^c)^{-1}, \quad (12)$$

$$G_{\tau+1}^s = -G\,(G_1^c)^{-1}, \quad (13)$$

conditions (3) are satisfied.

The common feature of Theorems 1, 2 and 4 suggests the method of filters designing. Some coefficients of composition filters must be assumed and the relations (7), (12) and (13) enable us to compute some parts of separation filters. The remaining values of filters coefficients are equal to zero. It is possible to proceed in an opposite direction, some parts of separation filters are assumed and the parameters of composition filters are computed from relations (8) and (10). Algorithms presented below result from the Theorems 1, 2 and 3. These theorems give open-loop computation schemes which can be immediately applied to designing transmultiplexer filters. Theorem 3 gives conditions much more complicated in a closed-loop form.

Algorithm 1.

According to Theorem 1, matrices G_1^c and G_τ^s must be non-singular. Let us consider a specific case when the transmultiplexer system has a delay $\tau = 1$. We obtain a simple algorithm for filter designing:

- arbitrarily choose matrix G_1^c (i.e. coefficients of composition filters),
- use formula $G_1^s = (G_1^c)^{-1}$ to compute the coefficients of separation filters (5) and take $h_k^s(0) = 0$ for all $1 \leq k \leq M$.

To provide the integer calculations it is sufficient to take integer G_1^c coefficients and fulfil condition $\det G_1^c = 1$.

Algorithm 2.

Theorem 2 gives the following algorithm:

- choose $m \geq 2$ and delay $m+1 \leq \tau \leq 2m-1$,
- take an arbitrary, non-singular matrix G_m^c,
- take zero values for the remaining parameters of the composition filters, i.e. $G_1^c = \ldots = G_{m-1}^c = 0$,
- compute the coefficients of separation filters $G_{\tau-m+1}^s = (G_m^c)^{-1}$,
- take zero values for the remaining parameters of the separation filters, i.e. $G_1^s = \ldots = G_{\tau-m}^s = 0$.

Similarly as in the above algorithm, to provide the integer calculations it is sufficient to take integer coefficients G_m^c and fulfil condition $\det G_m^c = 1$.

Algorithm 3.

Consequently to Theorem 4 matrix G_1^{c} must be non-singular. The algorithm of filter designing consists of the following steps:

- choose delay $1 \le \tau \le M$,
- take an arbitrary non-singular matrix G_1^{c},
- compute the remaining coefficient of composition filters $G_k^{\mathrm{c}} = G_{k-1}^{\mathrm{c}} G$, where $2 \le k \le M$,
- compute the coefficients of separation filters: $G_1^{\mathrm{s}} = \ldots = G_{\tau-1}^{\mathrm{s}} = 0$, $G_\tau^{\mathrm{s}} = (G_1^{\mathrm{c}})^{-1}$, $G_{\tau+1}^{\mathrm{s}} = -G\, G_\tau^{\mathrm{s}}$.

To provide the integer calculations it is sufficient to take integer values in G_1^{c} and $\det G_1^{\mathrm{c}} = 1$.

IV. Some Examples

Algorithm 1 under the assumption

$$G_1^{\mathrm{c}} = \begin{bmatrix} 1 & -1 & 2 & -2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (14)$$

gives the following composition filters

$$h_1^{\mathrm{c}} = \begin{bmatrix} -2 \\ 2 \\ -1 \\ 1 \end{bmatrix} \quad h_2^{\mathrm{c}} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \quad h_3^{\mathrm{c}} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad h_4^{\mathrm{c}} = 1$$

and separation filters

$$h_1^{\mathrm{s}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad h_2^{\mathrm{s}} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad h_3^{\mathrm{s}} = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} \quad h_4^{\mathrm{s}} = \begin{bmatrix} 0 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}.$$

Algorithm 2 for $m = 2$ and delay $\tau = 2$ under the assumption

$$G_2^{\mathrm{c}} = \begin{bmatrix} 1 & -1 & 2 & -2 \\ -1 & 2 & -3 & 4 \\ 2 & -3 & 6 & -7 \\ -2 & 4 & -7 & 10 \end{bmatrix}$$

gives the following composition filters

$$h_1^{\mathrm{c}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -2 \\ 2 \\ -1 \\ 1 \end{bmatrix} \quad h_2^{\mathrm{c}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ -3 \\ 2 \\ -1 \end{bmatrix} \quad h_3^{\mathrm{c}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -7 \\ 6 \\ -3 \\ 2 \end{bmatrix} \quad h_4^{\mathrm{c}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 10 \\ -7 \\ 4 \\ -2 \end{bmatrix}$$

and the separation filters

$$h_1^{\mathrm{c}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 1 \\ -2 \\ -1 \end{bmatrix} \quad h_2^{\mathrm{c}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 3 \\ 0 \\ -1 \end{bmatrix} \quad h_3^{\mathrm{c}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \quad h_4^{\mathrm{c}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}.$$

Algorithm 3 for delay $\tau = 2$ under the assumption (14) gives the following composition filters

$$h_1^{\mathrm{c}} = \begin{bmatrix} -2 \\ 2 \\ -1 \\ 1 \\ 2 \\ -1 \\ 1 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad h_2^{\mathrm{c}} = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad h_3^{\mathrm{c}} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad h_4^{\mathrm{c}} = 1$$

and separation filters

$$h_1^s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad h_2^s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad h_3^s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 1 \\ 1 \\ 0 \\ -1 \\ -1 \end{bmatrix} \quad h_4^s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}.$$

All examples present integer-to-integer filter banks. Integer values (14) for the beginning parts of the composite filter coefficients were assumed, the integer values were obtained for the separation filter coefficients and the remaining parts of the composition filters.

V. Conclusion

For the future next-generation telecommunications, the goals are the high data rates and customisable applications enabled by flexible technologies. Carefully designed transmultiplexers are able to fulfil such needs. Wideband wireless communications techniques have many advantages, however, there are a number of challenges and new possibilities in that area [9].

Transmultiplexer is a multiple-access system which arranges an easy adaptation and co-operation with existing telecommunications systems. It assimilates the idea of an intelligent network and reprogrammable electronic devices. Different linear systems can be introduced by a change of filter coefficients only, without hardware modifications. Filtering, upsampling and downsampling can be implemented in any kind of digital equipment. The use of integer-to-integer filter banks increases the number of possible applications of transmultiplexers. Methods enabling the calculation of integer coefficients for perfect reconstruction filters are particularly useful. Transmultiplexing systems equipped with such filters process signals using only the operations of multiplication and addition or subtractions of integers. Therefore, there are no computation errors such as rounding of numbers that occur in operations on real numbers. Owing to this, transmultiplexing systems equipped with integer filters can be used not only to transmit multimedia signals but also for encrypted data, lossless compressed signals or for computer software data where a change in even one single bit is inadmissible.

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